

Linear Systems on Graphs

Controllability of Structured Networks

Course Introduction

Orientation

Course Information · How It Runs · Weekly Plan

Fall 2026 · Graduate Program in Intelligent Robot Engineering
Department of Control & Instrumentation Engineering
Pukyong National University, Busan, Korea
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Orientation. Course Information

Course Linear System (A05470, section 001)

Credits 3 (3-3-0) · graduate elective

Time Wednesday, periods 7–9 (15:00–18:00)

Room B21-219

Instructor Nam-Jin Park / Control & Instrumentation Eng.

Office hours by e-mail appointment pnj@pknu.ac.kr

Course page <https://pknu-507.github.io/MARS/>

MARS Lab — slides and lecture notes are posted here

[Course goal] Master linear system theory from the **state-space viewpoint** and extend it to **networks**.

Decide and design the controllability of networks with unknown weights **from the graph alone**.

Orientation. Textbook and Tools

► Main text — lecture notes by the instructor

- N.-J. Park, *Linear Systems on Graphs: Controllability of Structured Networks*
- Three parts, ten chapters, plus proof appendices A · B
- Free PDF on the course page — <https://pknu-507.github.io/MARS/>
- Text and slides in English; lectures delivered in Korean

► Reference

- C.-T. Chen, *Linear System Theory and Design*, 4th ed.

► Tool — MATLAB only (free under the PKNU site license)

- Rank, eigenvalues, Gramians · digraph for reachability and matchings · symbolic determinants (Symbolic Math Toolbox)

► Prerequisites

- Linear algebra · differential equations · basic programming

[In one line] The first half **trusts the numbers** in A ; the second asks what can be certified when only the **zero/non-zero pattern** is known.

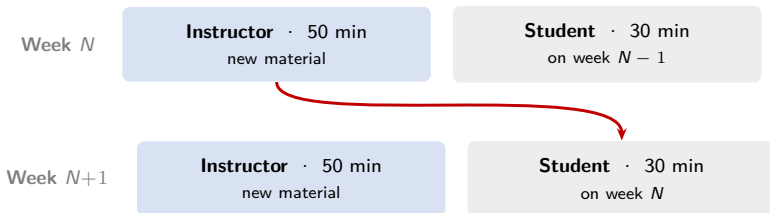
Orientation. How This Course Runs



- ▶ **First half** (weeks 1–7) — instructor lectures, Ch.1–Ch.6: **basic concepts, graph vocabulary, and the state-space** → **graph correspondence**. Deep proofs are left out.
- ▶ **Midterm** (week 8) — written exam, Ch.1–Ch.6.
- ▶ **Second half** (weeks 9–14) — each week opens with a **50-minute lecture**; a student then re-presents part of the **previous** week's material.
- ▶ **Week 16** — individual talks on a **paper each student picks** from a list handed out in week 9.

[The point] Learn **how to read** in the first half; **read and present** on your own in the second.

Orientation. Presentation Format (Weeks 9–14)

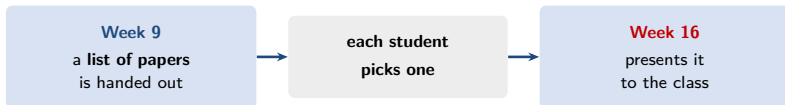


what the instructor covers in one week, a student re-presents in the next

- ▶ **50 min** — the instructor states the key definitions and theorems of the week.
- ▶ **30 min** — a student re-presents part of it: **20 min talk + 10 min Q & A**.
- ▶ Speakers and topics are announced in **week 9**.

[Why present] You find out what you did *not* understand only when you have to **say it back**.

Orientation. Term Project (Week 16)



- ▶ The list comes **right after the midterm**, so you choose knowing what the course has already covered.
- ▶ Talk length, report format, and what to cover are announced in **week 9**.

[Intent] Can you **read an unfamiliar paper on your own** with one semester's worth of tools?

Orientation. Evaluation

Item	Attendance	Midterm	Term project
Weight	10 %	40 %	50 %
When · scope	all weeks	week 8 · Ch.1–Ch.6	week 16 · paper talk

- ▶ **Midterm** — week 8, written: stating definitions, running the tests, constructing counterexamples.
- ▶ **Term project** — the week-16 paper talk and the technical report.
- ▶ Presentation work in weeks 9–14 counts **within the project score**.

[Attendance] Missing $\frac{1}{4}$ or more of the class hours results in an **F** under university regulations.

Orientation. Curriculum Map (I)

► Part I Linear Systems Theory

lectures

Ch. 1 Introduction: Systems, Models, and the Plan *a model is not the thing* wk 1

Ch. 2 Foundations: Models, Graphs, and Dynamics *the system as a graph* wk 2–3

Ch. 3 Controllability and Observability *Kalman rank · PBH* wk 4

► Part II Controllability of Structured Networks

lectures wk 5–7 · talks wk 9–12

Ch. 4 Structured Networks and Pattern Graphs *erase the numbers* wk 5

Ch. 5 SC and SSC: Determinant View *determinant trichotomy* wk 6

Ch. 6 Two Controllable Subspaces *how much survives* wk 7

Ch. 7 Whole-Graph Criteria *Lin · Mayeda · zero forcing* wk 9–10

Ch. 8 Node-Level Controllability: Fixed Nodes *which nodes decide* wk 11–12

[Turning point] Part I trusts **every number** in A — Part II keeps only its **zero / non-zero pattern**.

Orientation. Curriculum Map (II)

► Part III Input Placement: Synthesis

student talks

Ch. 9 Minimum Input Problem *how many inputs*

wk 13

Ch. 10 Optimal Input Placement and Outlook *where inputs go*

wk 14

► Appendix Proofs

deep-dive topics

Ch. A Proofs: Structural Side *Hosoe · Lin*

optional

Ch. B Proofs: Strong Structural Side *Mayeda · zero forcing*

optional

► Milestones

■ Midterm (written)

wk 8

■ Paper list handed out · speakers announced

wk 9

■ Student talks begin

wk 10

■ Term-project talks

wk 16

[In one line] From **deciding** (Part II) to **designing** (Part III) — and then to the literature.

Orientation. Weekly Schedule

First half wk 1–8 lectures				Second half wk 9–16 seminar			
Wk	Date	Topic	Note	Wk	Date	Topic	Note
1	9/2	Orientation · Ch.1 Introduction		9	10/28	Ch.7 (I) Lin · matching	papers
2	9/9	Ch.2 (I) Models and graphs		10	11/4	Ch.7 (II) Mayeda · zero forcing	talk
3	9/16	Ch.2 (II) Solutions · stability		11	11/11	Ch.8 (I) Fixed subspaces, nodes	talk
4	9/23	Ch.3 Controllability · PBH		12	11/18	Ch.8 (II) Input-addition test	talk
5	9/30	Ch.4 Pattern graphs · vocabulary		13	11/25	Ch.9 Minimum input problem	talk
6	10/7	Ch.5 Determinant view · SC, SSC		14	12/2	Ch.10 Optimal input placement	talk
7	10/14	Ch.6 Two controllable subspaces		15	12/9	Make-up week	
8	10/21	Midterm	Ch.1–6	16	12/16	Term-project talks	

[Academic calendar] midterm period **10/20–26** · make-up week **12/8–11** ·
 final period **12/14–20** **[No class]**
10/28 (Wed) official trip — made up in the make-up week

Orientation. What to Prepare

► This week

- Download the lecture-note PDF — skim **Ch.1** and **Ch.2**
- Install **MATLAB** — free for students under the **PKNU** site license

► Week 2 (9/9) — Ch.2 State-Space Models and Their Graphs

- One question will run through the whole semester:
*"If you do not know the numbers in A — only **where it is zero** and **where it is not** — can you control this network?"*
- Answering it needs, first, a way to **move a system onto a graph**.

[The promise] By the end of this course you can **read and present a paper on structural controllability** on your own.

Thank you

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